Electron Interference Effect

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"There are two entities in the universe that Man cannot comprehend:
Women - and the Dirac equation. And I am not so sure about women..."

Elementary derivation of the Dirac equation

Consider the wave equation in one dimension:

(1) $(\partial_t^2 - \partial_x^2)\Psi = (\partial_t - \partial_x)(\partial_t + \partial_x)\Psi = 0$

This is already the product of two Dirac equations

(2a) $(\partial_t + \partial_x)\Psi = 0$

(2b) $(\partial_t - \partial_x)\Psi = 0$

whose solutions are right and left going waves at speed c (=1).
If we identify a left-going wave with a state Ψ_1 and the right-going wave with a state Ψ_2 , we already get a two-state Dirac equation

(3) $(i\partial_t - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} i\partial_x)\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = 0$

We suppose that two states are enough.
When we turn a left-going wave adiabatically into a perpendicular (Y or Z) direction, the resulting state will be a linear superposition of our two states with equal weight for each state for symmetry reasons. Which means that the equation of motion in the Y-direction will read like this:

(4) $(i\partial_t - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} i\partial_y)\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = 0$

And the equation of motion for waves traveling in an arbitrary direction will be

(5) $(i\partial_t - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} i\partial_x - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} i\partial_y - \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} i\partial_z)\begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = 0$

which is, of course, our massless Dirac equation. Note that a single state cannot be a standing wave.

Adiabatic change is a physical process: we can funnel a traveling wave into a waveguide which is bent with a great radius of curvature. Such an operation can bring out the intricate nature of the SU(2) group with the Pauli matrices as generators just derived.
When we make a full 360° turn, we transform the state Ψ_1 into (- Ψ_1).
When we only make a 180° turn, this group can distinguish between a left and right turn, giving
$\Psi_1 \xrightarrow[\pm 180°]{} \pm i\,\Psi_2$
The addition of a mass does not change this situation qualitatively. It "only" brings in a second set of two states that oscillates with the first.

Now we do the following experiment (figure 1):
Split an incoming electron beam into two beam arms. Turn each arm back 180°, one left-handed, the other one right-handed in the same plane by electrostatic means (no B-field!)
Both arms join at the exit and the two partial waves can interfere and will be detected at (D).
According to what we just saw above, the two partial waves will annul each other, leading to total reflection. We only get a measured intensity when one of the arms is blocked.

figure 1: electron interference